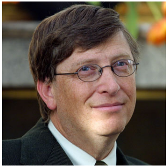
Predicate Abstraction with SATABS

SATABS

Version 1.0, 2010

Outline



> *"Things like even software verification, this has been the Holy Grail of computer science for many decades, but now in some very key areas, for example, driver verification we're building tools that can do* **actual proof** *about the software and how it works in order to guarantee the reliability."*

Bill Gates, April 18, 2002
Keynote address at WinHec 2002

"*One of the least visible ways that Microsoft Research contributed to Vista, but something I like to talk about, is the work we did on what's called the Static Driver Verifier. People who develop device drivers for Vista can verify the properties of their drivers before they ever even attempt to test that. What's great about this technology is there is no testing involved. For the properties that it is proving, they are either true or false.*
You don't have to ask yourself
"Did I come up with a good test case or not?""

Rick Rashid, Microsoft Research chief
father of CMU's Mach Operating System (Mac OS X)
news.cnet.com interview, 2008

Model Checking with Predicate Abstraction

SATABS

- A heavy-weight formal analysis technique

- Recent successes in software verification, e.g., SLAM at Microsoft

- The abstraction reduces the size of the model by removing irrelevant detail

Model Checking with Predicate Abstraction

- Goal: make the abstract model small enough for an analysis with a BDD-based Model Checker

- Idea: only track predicates on data,
  and remove data variables from model

- Mostly works with control-flow dominated properties

Notation for Abstractions

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Abstract Domain

Approximate representation of
sets of concrete values

$$S \underset{\gamma}{\overset{\alpha}{\rightleftarrows}} \hat{S}$$

Predicate Abstraction as Abstract Domain

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- We are given a set of predicates over S, denoted by $\Pi_1, \ldots, \Pi_n$.

- An abstract state is a valuation of the predicates:

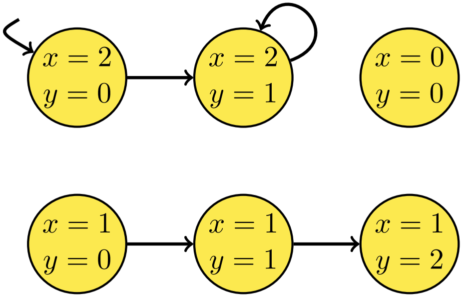
$$\hat{S} = \mathbb{B}^n$$

- The abstraction function:

$$\alpha(s) = \langle \Pi_1(s), \ldots, \Pi_n(s) \rangle$$

Predicate Abstraction: the Basic Idea

Concrete states over variables x, y:



Predicate Abstraction: the Basic Idea

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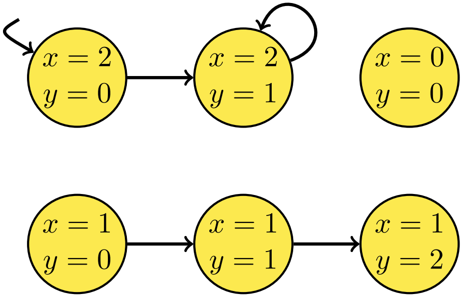

Predicates:

$$p_1 \iff x > y$$
$$p_2 \iff y = 0$$

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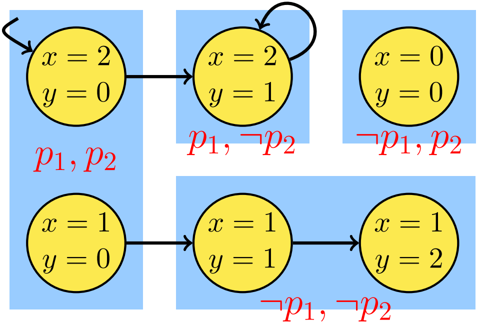

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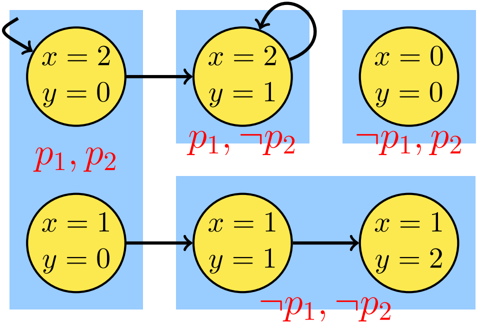

Predicates:

$$p_1 \iff x > y$$
$$p_2 \iff y = 0$$

Abstract Transitions?

Existential Abstraction[1]

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Definition (Existential Abstraction)

A model $\hat{M} = (\hat{S}, \hat{S}_0, \hat{T})$ is an *existential abstraction* of $M = (S, S_0, T)$ with respect to $\alpha : S \to \hat{S}$ iff

- $\exists s \in S_0.\, \alpha(s) = \hat{s} \quad \Rightarrow \quad \hat{s} \in \hat{S}_0$ and
- $\exists (s, s') \in T.\, \alpha(s) = \hat{s} \wedge \alpha(s') = \hat{s}' \quad \Rightarrow \quad (\hat{s}, \hat{s}') \in \hat{T}$.

[1] Clarke, Grumberg, Long: *Model Checking and Abstraction*, ACM TOPLAS, 1994

Minimal Existential Abstractions

There are obviously many choices for an existential abstraction for a given α.

Definition (Minimal Existential Abstraction)

A model $\hat{M} = (\hat{S}, \hat{S}_0, \hat{T})$ is the *minimal existential abstraction* of $M = (S, S_0, T)$ with respect to $\alpha : S \to \hat{S}$ iff

- $\exists s \in S_0 . \alpha(s) = \hat{s} \iff \hat{s} \in \hat{S}_0$ and
- $\exists (s, s') \in T . \alpha(s) = \hat{s} \wedge \alpha(s') = \hat{s}' \iff (\hat{s}, \hat{s}') \in \hat{T}$.

This is the most precise existential abstraction.

Existential Abstraction

We write $\alpha(\pi)$ for the abstraction of a path $\pi = s_0, s_1, \ldots$:

$$\alpha(\pi) \; = \; \alpha(s_0), \alpha(s_1), \ldots$$

Existential Abstraction

SATABS

We write $\alpha(\pi)$ for the abstraction of a path $\pi = s_0, s_1, \ldots$:

$$\alpha(\pi) \;=\; \alpha(s_0), \alpha(s_1), \ldots$$

Lemma

Let $\hat{M}$ be an existential abstraction of M. The abstraction of every path (trace) π in M is a path (trace) in $\hat{M}$.

$$\pi \in M \quad \Rightarrow \quad \alpha(\pi) \in \hat{M}$$

Proof by induction.
We say that $\hat{M}$ overapproximates M.

Abstracting Properties

Reminder: we are using

- a set of atomic propositions (predicates) A, and
- a state-labelling function $L : S \rightarrow \mathscr{P}(A)$

in order to define the meaning of propositions in our properties.

We define an abstract version of it as follows:

- First of all, the negations are pushed into the atomic propositions.
  E.g., we will have

$$x = 0 \quad \in A$$

and

$$x \neq 0 \quad \in A$$

Abstracting Properties

- An abstract state $\hat{s}$ is labelled with $a \in A$ iff all of the corresponding concrete states are labelled with a.

$$a \in \hat{L}(\hat{s}) \quad \Longleftrightarrow \quad \forall s | \alpha(s) = \hat{s}.\, a \in L(s)$$

- This also means that an abstract state may have neither the label $x = 0$ nor the label $x \neq 0$ – this may happen if it concretizes to concrete states with different labels!

Conservative Abstraction

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The keystone is that existential abstraction is conservative for certain properties:

Theorem (Clarke/Grumberg/Long 1994)

Let ϕ be a $\forall$CTL formula where all negations are pushed into the atomic propositions, and let $\hat{M}$ be an existential abstraction of M. If ϕ holds on $\hat{M}$, then it also holds on M.*

$$\hat{M} \models \phi \quad \Rightarrow \quad M \models \phi$$

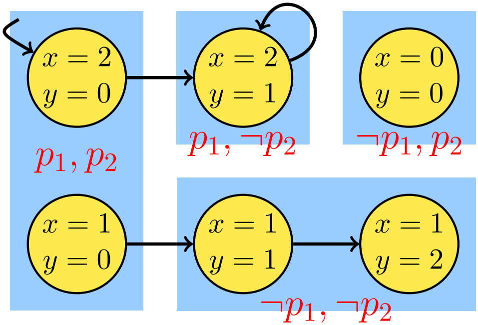
We say that an existential abstraction is conservative for $\forall$CTL* properties. The same result can be obtained for LTL properties.

The proof uses the lemma and is by induction on the structure of ϕ. The converse usually does not hold.

Conservative Abstraction

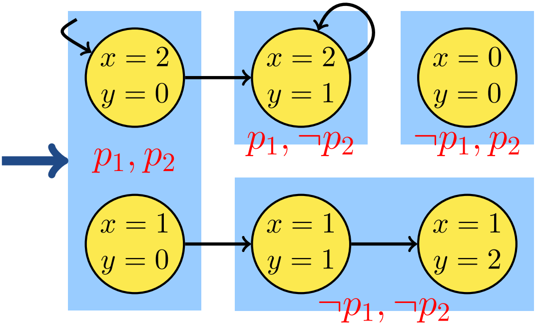
We hope: computing $\hat{M}$ and checking $\hat{M} \models \phi$ is easier than checking $M \models \phi$.

Back to the Example



Back to the Example

SATABS



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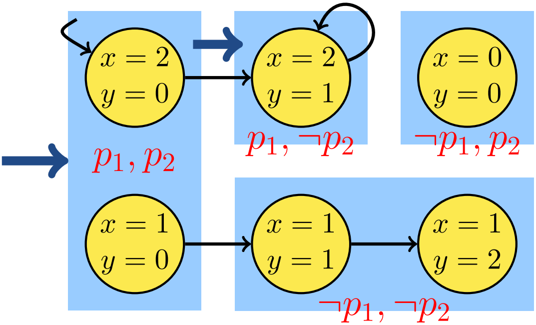

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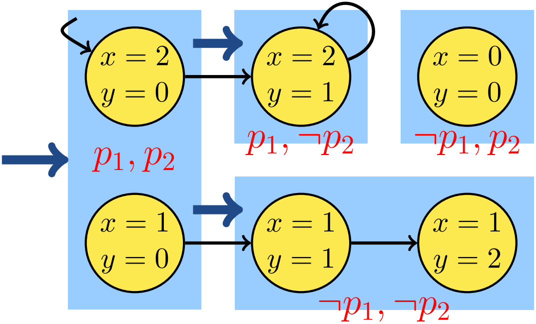

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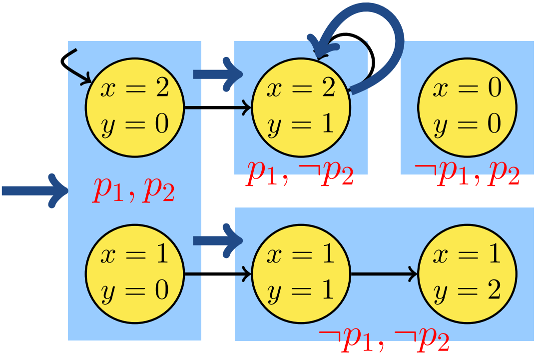

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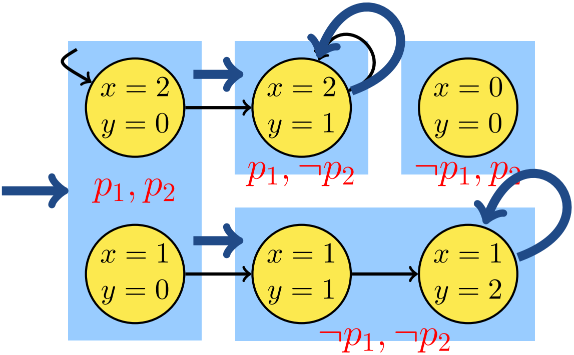
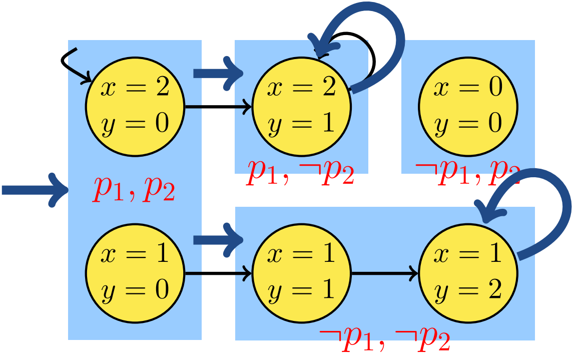


Let's try a Property

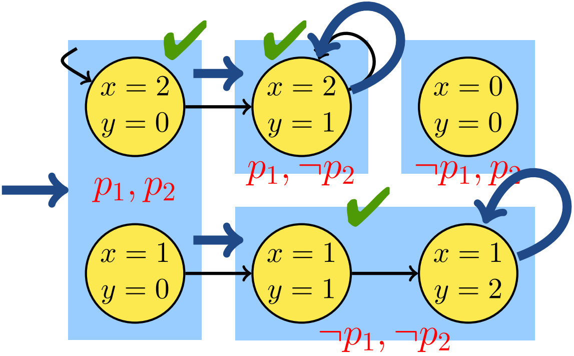


Property:

$$x > y \vee y \neq 0 \qquad \Longleftrightarrow \qquad p_1 \vee \neg p_2$$

Let's try a Property

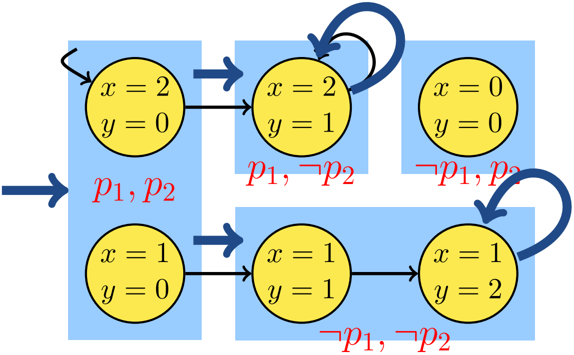
SATABS



Property:

$$x > y \lor y \neq 0 \quad \Longleftrightarrow \quad p_1 \lor \neg p_2$$

Another Property



Property:

$x > y \iff p_1$

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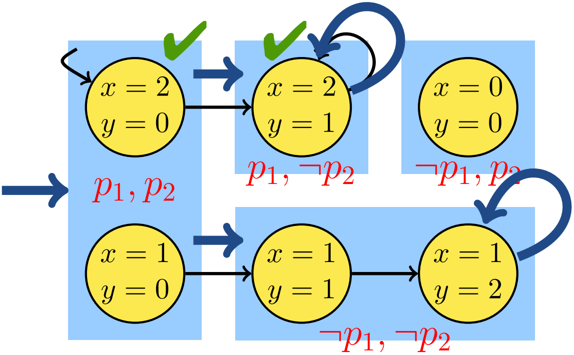

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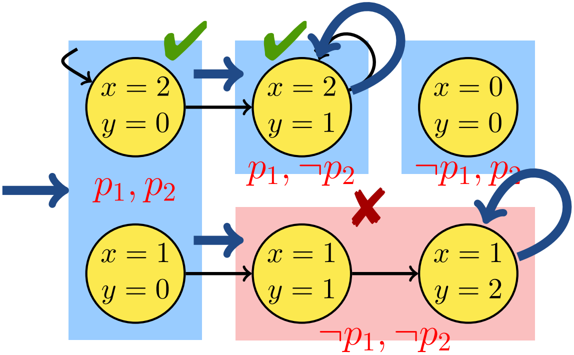

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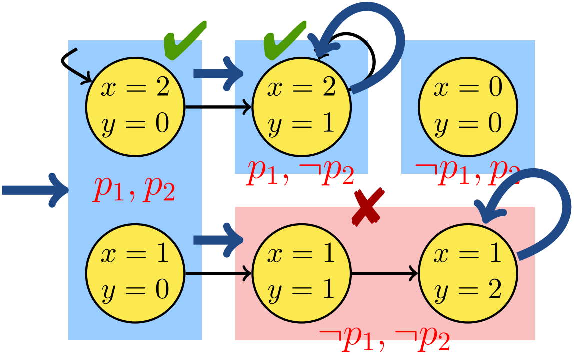


Property:

$x > y \iff p_1$

But: the counterexample is spurious

SLAM

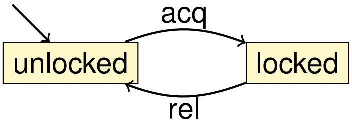
- Microsoft blames most Windows crashes on third party device drivers
- The Windows device driver API is quite complicated
- Drivers are low level C code
- SLAM: Tool to automatically check device drivers for certain errors
- SLAM is shipped with Device Driver Development Kit
- Full detail available at `http://research.microsoft.com/slam/`

SLIC

- Finite state language for defining properties
  - Monitors behavior of C code
  - Temporal safety properties (security automata)
  - familiar C syntax

- Suitable for expressing control-dominated properties
  - e.g., proper sequence of events
  - can track data values

SLIC Example

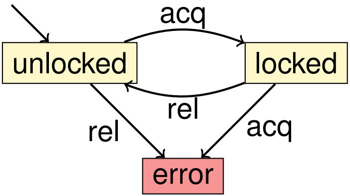



```
state {
  enum {Locked, Unlocked}
    s = Unlocked;
}

KeAcquireSpinLock.entry {
   if (s==Locked) abort;
   else s = Locked;
}

KeReleaseSpinLock.entry {
   if (s==Unlocked) abort;
   else s = Unlocked;
}
```

SLIC Example

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  if (s==Unlocked) abort;
  else s = Unlocked;
}
```

Refinement Example

SATABS

```
do {
    KeAcquireSpinLock ();
    nPacketsOld = nPackets;
    if (request) {
        request = request->Next;
        KeReleaseSpinLock ();
        nPackets++;
    }
} while(nPackets != nPacketsOld);

KeReleaseSpinLock ();
```

Refinement Example

SATABS

Does this code obey the locking rule?

```
do {
    KeAcquireSpinLock();
    nPacketsOld = nPackets;
    if (request) {
        request = request->Next;
        KeReleaseSpinLock();
        nPackets++;
    }
} while(nPackets != nPacketsOld);

KeReleaseSpinLock();
```

Refinement Example

```
do {
    KeAcquireSpinLock ();

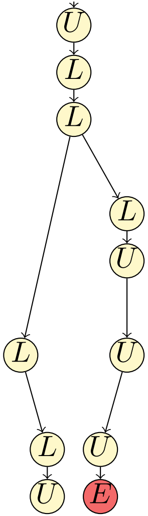
    if (*)  {

        KeReleaseSpinLock ();

    }
} while(*);

KeReleaseSpinLock ();
```

Refinement Example

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do {
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    }
} while(*);

KeReleaseSpinLock();
```
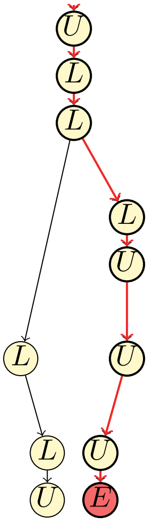

Refinement Example

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do {
    KeAcquireSpinLock ();

    if (*)  {

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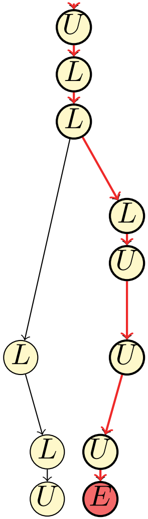
    }
} while(*);

KeReleaseSpinLock ();
```

Refinement Example

SATABS



do {

KeAcquireSpinLock ();

if ($*$) {

KeReleaseSpinLock ();

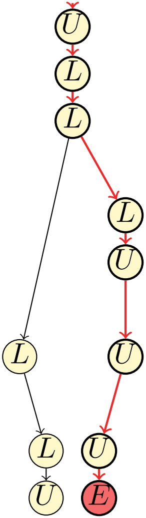
}

} **while**($*$);

KeReleaseSpinLock ();

Is this path concretizable?

Refinement Example

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KeReleaseSpinLock();
```

Refinement Example

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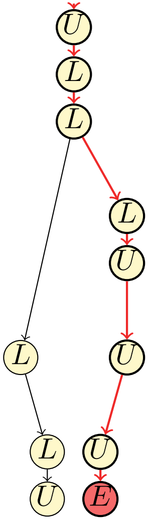


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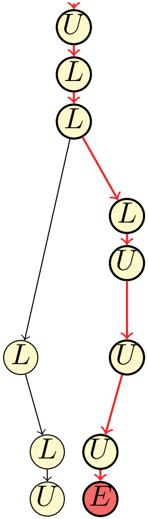
} while(nPackets != nPacketsOld);

KeReleaseSpinLock();
```

This path is spurious!

Refinement Example



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KeReleaseSpinLock();
```
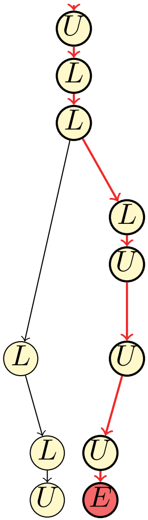

Let's add the predicate nPacketsOld==nPackets

Refinement Example

SATABS




```
do {
    KeAcquireSpinLock();
    nPacketsOld = nPackets;          b=true;
    if(request) {
        request = request->Next;
        KeReleaseSpinLock();
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} while(nPackets != nPacketsOld);

KeReleaseSpinLock();
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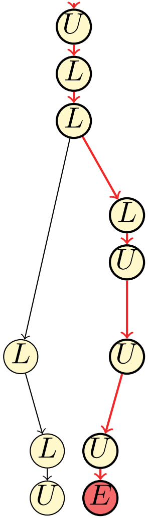


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            KeReleaseSpinLock ();
            nPackets++;                  b=b?false:∗;
      }
} while(nPackets != nPacketsOld);   !b

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Let's add the predicate nPacketsOld==nPackets

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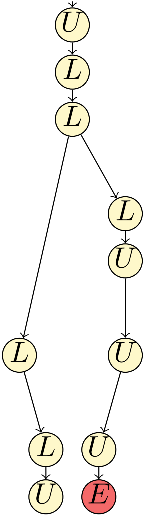


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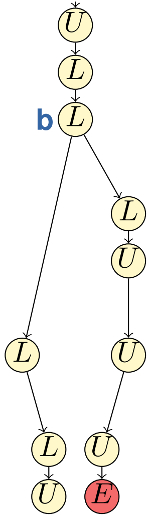


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Refinement Example

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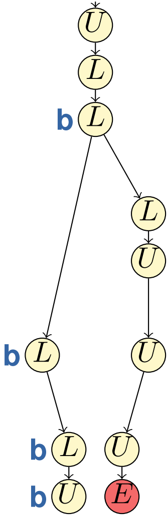


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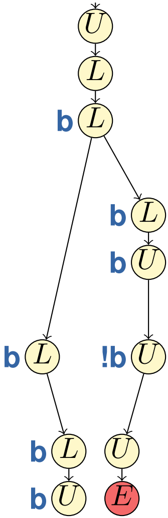


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Refinement Example

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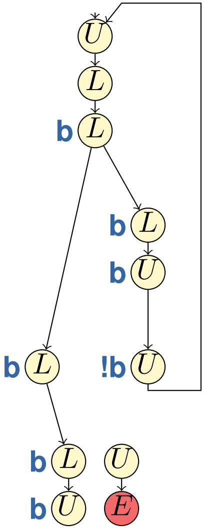


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Refinement Example

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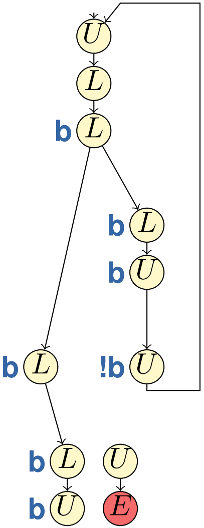


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      KeAcquireSpinLock();
      b=true;
      if (*) {

            KeReleaseSpinLock();
            b=b?false:*;
      }
} while( !b );

KeReleaseSpinLock();
```

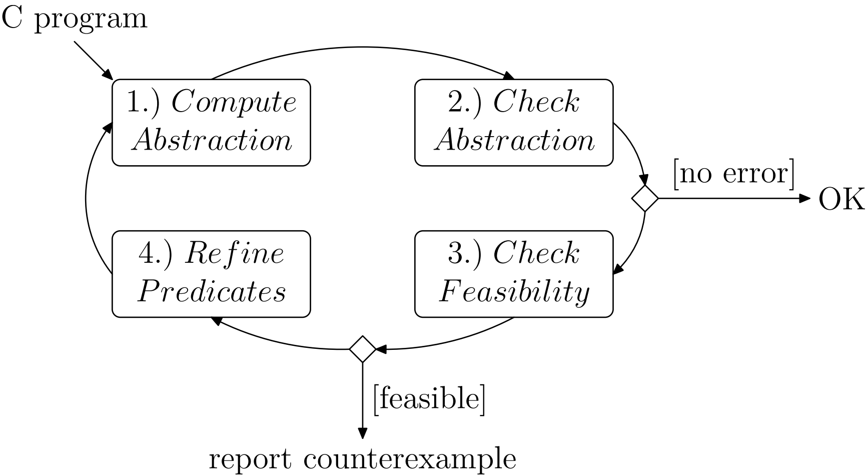
The property holds!

Counterexample-guided Abstraction Refinement

- ”CEGAR”
- An iterative method to compute a sufficiently precise abstraction
- Initially applied in the context of hardware [Kurshan]

CEGAR Overview

SATABS



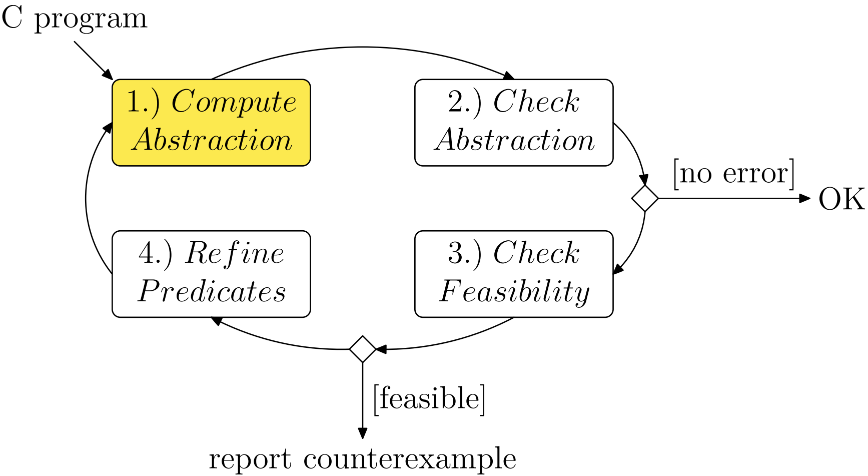
Counterexample-guided Abstraction Refinement

Claims:

1. This never returns a false error.
2. This never returns a false proof.

3. This is complete for finite-state models.
4. But: no termination guarantee in case of infinite-state systems

Computing Existential Abstractions of Programs



Computing Existential Abstractions of Programs

```
int main() {
  int i;

  i=0;

  while(even(i))
    i++;
}
```

C Program

Computing Existential Abstractions of Programs

SATABS

```
int main() {
  int i;

  i=0;

  while(even(i))
    i++;
}
```

+

$$p_1 \iff i = 0$$
$$p_2 \iff even(i)$$

C Program   Predicates

Computing Existential Abstractions of Programs

SATABS

```
int main() {
  int i;

  i=0;

  while(even(i))
    i++;
}
```

+

$$p_1 \iff i = 0$$
$$p_2 \iff even(i)$$

→

```
void main() {
  bool p1, p2;

  p1=TRUE;
  p2=TRUE;

  while(p2) {
    p1= p1 ? FALSE : *;
    p2= !p2;
  }
}
```

C Program — Predicates — Boolean Program

Computing Existential Abstractions of Programs

SATABS

```
int main() {
  int i;

  i=0;

  while(even(i))
    i++;
}
```

C Program

+

$$p_1 \iff i = 0$$
$$p_2 \iff even(i)$$

Predicates

```
void main() {
  bool p1, p2;

  p1=TRUE;
  p2=TRUE;

  while(p2) {
    p1= p1 ? FALSE : *;
    p2= !p2;
  }
}
```

Boolean Program

Minimal?

Predicate Images

Reminder:

$$Image(X) = \{s' \in S \mid \exists s \in X.\, T(s, s')\}$$

We need

$$\widehat{Image}(\hat{X}) = \{\hat{s}' \in \hat{S} \mid \exists \hat{s} \in \hat{X}.\, \hat{T}(\hat{s}, \hat{s}')\}$$

$\widehat{Image}(\hat{X})$ is equivalent to

$$\{\hat{s}, \hat{s}' \in \hat{S}^2 \mid \exists s, s' \in S^2.\, \alpha(s) = \hat{s} \wedge \alpha(s') = \hat{s}' \wedge T(s, s')\}$$

This is called the predicate image of T.

Enumeration

- Let's take existential abstraction seriously
- Basic idea: with n predicates, there are $2^n \cdot 2^n$ possible abstract transitions
- Let's just check them!

Enumeration: Example

SATABS

Predicates

$$\begin{array}{lcl} p_1 & \iff & i = 1 \\ p_2 & \iff & i = 2 \\ p_3 & \iff & \mathsf{even}(i) \end{array}$$

Enumeration: Example

SATABS

Predicates

$$
\begin{aligned}
p_1 &\iff i = 1 \\
p_2 &\iff i = 2 \\
p_3 &\iff \mathsf{even}(i)
\end{aligned}
$$

Basic Block

```
i++;
```

Enumeration: Example

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \text{even}(i)$$

Basic Block

```
i++;
```

$\rightarrow$

T

$$i' = i + 1$$

Enumeration: Example

SATABS

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \text{even}(i)$$

Basic Block

```
i++;
```

$\rightarrow$

T

$$i' = i + 1$$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

p'_1	p'_2	p'_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Enumeration: Example

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \text{even}(i)$$

Basic Block

```
i++;
```

$\rightarrow$

T

$$i' = i + 1$$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$\xrightarrow{?}$

p_1'	p_2'	p_3'
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Enumeration: Example

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \mathrm{even}(i)$$

Basic Block

```
i++;
```

$\rightarrow$

T

$$i' = i + 1$$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$\xrightarrow{?}$

p'_1	p'_2	p'_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Query to Solver

$$i \neq 1 \wedge i \neq 2 \wedge \overline{\mathrm{even}(i)} \wedge$$
$$i' = i + 1 \wedge$$
$$i' \neq 1 \wedge i' \neq 2 \wedge \overline{\mathrm{even}(i')}$$

Enumeration: Example

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \text{even}(i)$$

Basic Block

```
i++;
```

$\rightarrow$

T

$$i' = i + 1$$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

p_1'	p_2'	p_3'
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Query to Solver

$$i \neq 1 \wedge i \neq 2 \wedge \overline{\text{even}(i)} \wedge$$
$$i' = i + 1 \wedge$$
$$i' \neq 1 \wedge i' \neq 2 \wedge \overline{\text{even}(i')}$$

Enumeration: Example

SATABS

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \text{even}(i)$$

Basic Block

```
i++;
```

$\rightarrow$

T

$$i' = i + 1$$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

?

p_1'	p_2'	p_3'
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Query to Solver

$$i \neq 1 \wedge i \neq 2 \wedge \overline{\text{even}(i)} \wedge$$
$$i' = i + 1 \wedge$$
$$i' \neq 1 \wedge i' \neq 2 \wedge \text{even}(i')$$

Enumeration: Example

SATABS

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \text{even}(i)$$

Basic Block

```
i++;
```

$\rightarrow$

T

$$i' = i + 1$$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

✔

p'_1	p'_2	p'_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Query to Solver

$$i \neq 1 \wedge i \neq 2 \wedge \overline{\text{even}(i)} \wedge$$
$$i' = i + 1 \wedge$$
$$i' \neq 1 \wedge i' \neq 2 \wedge \text{even}(i')$$

Enumeration: Example

SATABS

Predicates

$$p_1 \iff i = 1$$
$$p_2 \iff i = 2$$
$$p_3 \iff \text{even}(i)$$

Basic Block

```
i++;
```

→

T

$$i' = i + 1$$

p_1	p_2	p_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

p'_1	p'_2	p'_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Query to Solver

…and so on …

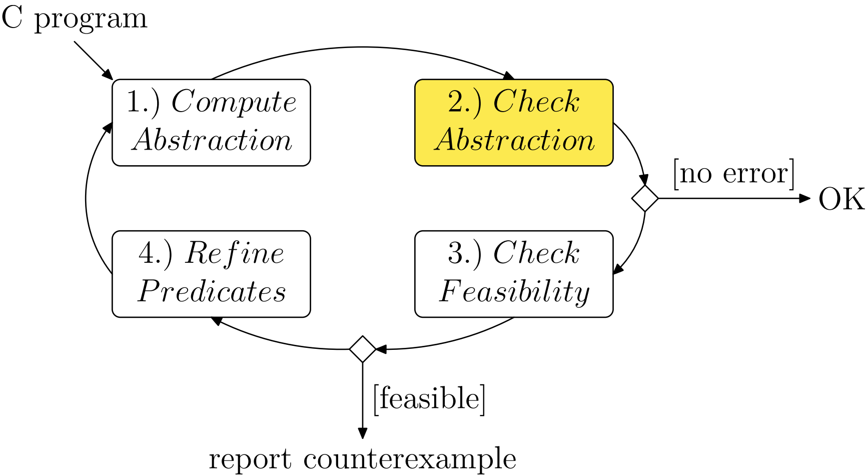
Predicate Images

SATABS

- ✘ Computing the minimal existential abstraction can be way too slow
- Use an over-approximation instead
  - ✔ Fast(er) to compute
  - ✘ But has additional transitions
- Examples:
  - Cartesian approximation (SLAM)
  - FastAbs (SLAM)
  - Lazy abstraction (Blast)
  - Predicate partitioning (VCEGAR)

Checking the Abstract Model

SATABS



Checking the Abstract Model

SATABS

- No more integers!
- But:
  - All control flow constructs, including function calls
  - (more) non-determinism

✔ BDD-based model checking now scales

Finite-State Model Checkers: SMV

SATABS

① Variables

```
VAR b0_argc_ge_1: boolean;               -- argc >= 1
VAR b1_argc_le_2147483646: boolean;      -- argc <= 2147483646
VAR b2: boolean;                         -- argv[argc] == NULL
VAR b3_nmemb_ge_r: boolean;              -- nmemb >= r
VAR b4: boolean;                         -- p1 == &array[0]
VAR b5_i_ge_8: boolean;                  -- i >= 8
VAR b6_i_ge_s: boolean;                  -- i >= s
VAR b7: boolean;                         -- 1 + i >= 8
VAR b8: boolean;                         -- 1 + i >= s
VAR b9_s_gt_0: boolean;                  -- s > 0
VAR b10_s_gt_1: boolean;                 -- s > 1
...
```

Finite-State Model Checkers: SMV

SATABS

② Control Flow

```
-- program counter: 56 is the "terminating" PC
VAR PC: 0..56;
ASSIGN init(PC):=0; -- initial PC

ASSIGN next(PC):=case
    PC=0: 1; -- other
    PC=1: 2; -- other
    . . .
    PC=19: case  -- goto (with guard)
      guard19: 26;
      1: 20;
    esac;
    . . .
```

Finite-State Model Checkers: SMV

③ Data

```
TRANS (PC=0) -> next(b0_argc_ge_1)=b0_argc_ge_1
             & next(b1_argc_le_213646)=b1_argc_le_21646
             & next(b2)=b2
             & (!b30 | b36)
             & (!b17 | !b30 | b42)
             & (!b30 | !b42 | b48)
             & (!b17 | !b30 | !b42 | b54)
             & (!b54 | b60)

TRANS (PC=1) -> next(b0_argc_ge_1)=b0_argc_ge_1
             & next(b1_argc_le_214646)=b1_argc_le_214746
             & next(b2)=b2
             & next(b3_nmemb_ge_r)=b3_nmemb_ge_r
             & next(b4)=b4
             & next(b5_i_ge_8)=b5_i_ge_8
             & next(b6_i_ge_s)=b6_i_ge_s
             ...
```

Finite-State Model Checkers: SMV

SATABS

④ Property

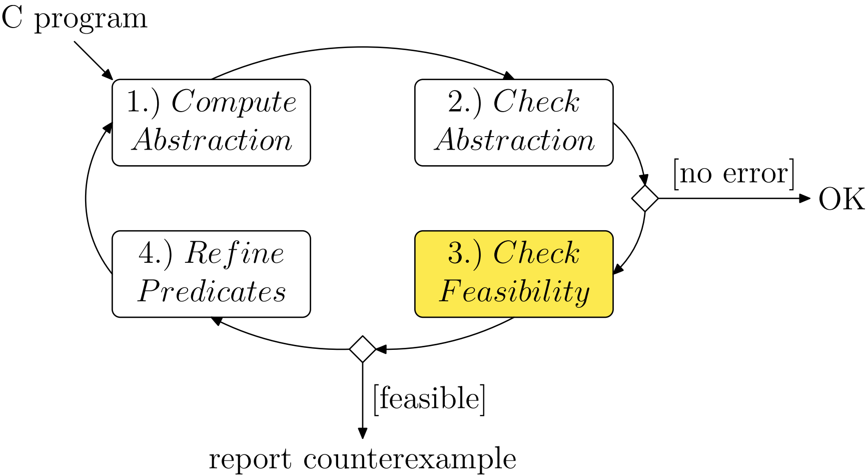
```
-- the specification

-- file main.c line 20 column 12
-- function c::very_buggy_function
SPEC AG ((PC=51) -> !b23)
```

Finite-State Model Checkers: SMV

- If the property holds, we can terminate
- If the property fails, SMV generates a counterexample with an assignment for all variables, including the PC

Simulating the Counterexample



Lazy Abstraction

SATABS

- The progress guarantee is only valid if the minimal existential abstraction is used.

- Thus, distinguish spurious transitions from spurious prefixes.

- Refine spurious transitions separately to obtain minimal existential abstraction
- SLAM: `Constrain`

Lazy Abstraction

- One more observation:
  each iteration only causes only minor changes in the abstract model

- Thus, use "incremental Model Checker", which retains the set of reachable states between iterations (BLAST)

Example Simulation

SATABS

```
int main() {
    int x, y;
    y=1;
    x=1;
    if (y>x)
        y--;
    else
        y++;
    assert(y>x);
}
```

Predicate:

$y > x$

```
main() {
    bool b0; // y>x
    b0=*;
    b0=*;
    if (b0)
        b0=*;
    else
        b0=*;
    assert(b0);
}
```

Example Simulation

```
int main() {
    int x, y;
    y=1;
    x=1;
    if (y>x)
        y−−;
    else
        y++;
    assert(y>x);
}
```

Predicate:

y>x

```
main() {
    bool b0; // y>x
    b0=*;
    b0=*;
    if (b0)
        b0=*;
    else
        b0=*;
    assert(b0);
}
```

Example Simulation

SATABS

```
int main() {
    int x, y;
  y=1;
  x=1;
  if (y>x)
      y--;
  else
      y++;
  assert(y>x);
}
```

Example Simulation

```
int main() {
    int x, y;
    y=1;
    x=1;
    if (y>x)
        y−−;
    else
        y++;
    assert(y>x);
}
```

We now do a path test, so convert to SSA.

Example Simulation

$\textbf{int}\ \mathrm{main}()\ \{$

$\quad \textbf{int}\ x,\ y;$

$\quad y_1 = 1;$

$\quad x_1 = 1;$

$\quad \textbf{if}\ (y_1 > x_1)$

$\qquad y_2 = y_1 - 1;$

$\quad \textbf{else}$

$\qquad y{+}{+};$

$\quad \mathrm{assert}(y_2 > x_1);$

$\}$

Example Simulation

int main() {

  int x, y;

  y_1=1;

  x_1=1;

  if (y_1>x_1)

    y_2=y_1−1;

  else

    y++;

  assert(y_2>x_1);

}

$$y_1 = 1 \quad \wedge$$
$$x_1 = 1 \quad \wedge$$
$$y_1 > x_1 \quad \wedge$$
$$y_2 = y_1 - 1 \quad \wedge$$

$$\neg(y_2 > x_0)$$

Example Simulation

SATABS

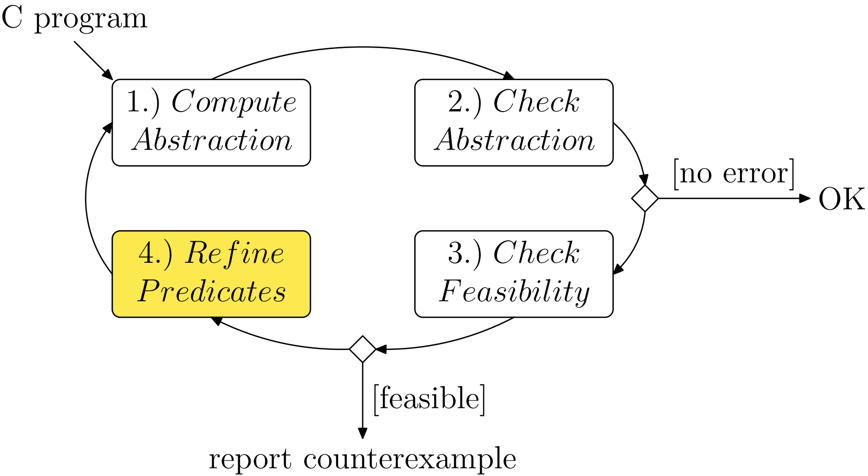
```
int main() {
    int x, y;
    y1=1;
    x1=1;
    if (y1>x1)
        y2=y1−1;
    else
        y++;
    assert(y2>x1);
}
```

$$
\begin{aligned}
&y_1 = 1 \quad \wedge \\
&x_1 = 1 \quad \wedge \\
&y_1 > x_1 \quad \wedge \\
&y_2 = y_1 - 1 \quad \wedge
\end{aligned}
$$

$$\neg(y_2 > x_0)$$

This is UNSAT, so $\hat{\pi}$ is spurious.

Refining the Abstraction



Manual Proof!

```
int main() {
    int x, y;
    y=1;

    x=1;

    if (y>x)
        y−−;
    else

        y++;

    assert(y>x);
}
```

Manual Proof!

SATABS

```
int main() {
    int x, y;
    y=1;
```

$\{y = 1\}$

```
    x=1;

    if (y>x)
        y--;
    else

        y++;

    assert(y>x);
}
```

Manual Proof!

SATABS

```
int main() {
    int x, y;
    y=1;
```
$\{y = 1\}$
```
    x=1;
```
$\{x = 1 \wedge y = 1\}$
```
    if (y>x)
        y--;
    else

        y++;

    assert(y>x);
}
```

Manual Proof!

```
int main() {
    int x, y;
    y=1;
```
$\{y = 1\}$
```
    x=1;
```
$\{x = 1 \wedge y = 1\}$
```
    if (y>x)
        y--;
    else
```
$\{x = 1 \wedge y = 1 \wedge \neg y > x\}$
```
        y++;


    assert(y>x);
}
```

Manual Proof!

```
int main() {
    int x, y;

    y=1;
```

$\{y = 1\}$

```
    x=1;
```

$\{x = 1 \wedge y = 1\}$

```
    if (y>x)
        y−−;
    else
```

$\{x = 1 \wedge y = 1 \wedge \neg y > x\}$

```
        y++;
```

$\{x = 1 \wedge y = 2 \wedge y > x\}$

```
    assert(y>x);
}
```

This proof uses strongest post-conditions

An Alternative Proof

SATABS

```
int main() {
    int x, y;
    y=1;

    x=1;

    if (y>x)
        y--;
    else

        y++;

    assert(y>x);
}
```

An Alternative Proof

SATABS

```
int main() {
    int x, y;
    y=1;

    x=1;

    if (y>x)
       y--;
    else

       y++;
    {y > x}
    assert(y>x);
}
```

An Alternative Proof

```
int main() {
    int x, y;
    y=1;

    x=1;

    if (y>x)
        y--;
    else
        {y + 1 > x}
        y++;
    {y > x}
    assert(y>x);
}
```

An Alternative Proof

SATABS

```
int main() {
    int x, y;
    y=1;


    x=1;
```
$\{\neg y > x \Rightarrow y + 1 > x\}$
```
    if (y>x)
        y−−;
    else
```
$\{y + 1 > x\}$
```
        y++;
```
$\{y > x\}$
```
    assert(y>x);
}
```

An Alternative Proof

```
int main() {
    int x, y;
    y=1;
```

$\{\neg y > 1 \Rightarrow y+1 > 1\}$

```
    x=1;
```

$\{\neg y > x \Rightarrow y+1 > x\}$

```
    if (y>x)
        y--;
    else
```

$\{y+1 > x\}$

```
        y++;
```

$\{y > x\}$

```
    assert(y>x);
}
```

An Alternative Proof

SATABS

```
int main() {
    int x, y;

    y=1;
```
$\{\neg y > 1 \Rightarrow y+1 > 1\}$
```
    x=1;
```
$\{\neg y > x \Rightarrow y+1 > x\}$
```
    if (y>x)
        y−−;
    else
```
$\{y+1 > x\}$
```
        y++;
```
$\{y > x\}$
```
    assert(y>x);
}
```

We are using weakest pre-conditions here

$wp(x\texttt{:=}E, P) = P[x/E]$

$wp(S\texttt{;}T, Q) = wp(S, wp(T, Q))$

$$wp(\texttt{if}(c)\ A\ \texttt{else}\ B, P) = (B \Rightarrow wp(A, P)) \wedge (\neg B \Rightarrow wp(B, P))$$

The proof for the ”true” branch is missing

Refinement Algorithms

SATABS

Using WP

1. Start with failed guard G
2. Compute $wp(G)$ along the path

Using SP

1. Start at beginning
2. Compute $sp(\ldots)$ along the path

- Both methods eliminate the trace
- Advantages/disadvantages?

Predicate Localization

SATABS

Example:

```
int x, y;

x=10;

y=x+10;

y=y+10;

assert(y==30);
```

original program

Predicate Localization

Example:

```
int x, y;

x=10;

y=x+10;

y=y+10;

assert(y==30);
```

original program

$+$

$$x = 10$$
$$y = 20$$
$$y = 30$$

predicates

Predicate Localization

Example:

```
int x, y;

x=10;

y=x+10;

y=y+10;

assert(y==30);
```

original program

$+$

$$x = 10$$
$$y = 20$$
$$y = 30$$

predicates

$=$

```
bool x10, y20, y30;

x10=1;

y20,y30=x10?1:∗,∗;

y20,y30=∗,y20?1:∗;

assert(y30);
```

abstraction

Predicate Localization

SATABS

Example:

```
int x, y;

x=10;
    {x = 10}
y=x+10;
    {y = 20}
y=y+10;
    {y = 30}
assert(y==30);
```

original program

$+$

$x = 10$
$y = 20$
$y = 30$

predicates

$=$

```
bool x10, y20, y30;

x10=1;

y20,y30=x10?1:∗,∗;

y20,y30=∗,y20?1:∗;

assert(y30);
```

abstraction

Predicate Localization

SATABS

Example:

```
int x, y;

x=10;
```
$\{x = 10\}$
```
y=x+10;
```
$\{y = 20\}$
```
y=y+10;
```
$\{y = 30\}$
```
assert(y==30);
```

original program

$+$

$x = 10$
$y = 20$
$y = 30$

predicates

$=$

```
bool x10, y20, y30;

x10=1;

y20,y30=x10?1:*,*;

y20,y30=*,y20?1:*;

assert(y30);
```

abstraction

We really only want to track specific predicates at each location!

Predicate Localization

SATABS

- Track a separate set of predicates for each location

- ✔ Makes predicate image easier
- ✔ Makes simulation of transitions easier
- ✔ Makes the check of the abstract model easier

Predicate Refinement for Paths

Recall the decision problem we build for simulating paths:

$$x_1 = 10 \quad \wedge \quad y_1 = x_1 + 10 \quad \wedge \quad y_2 = y_1 + 10 \quad \wedge \quad y_2 \neq 30$$

Predicate Refinement for Paths

SATABS

Recall the decision problem we build for simulating paths:

$$x_1 = 10 \quad \wedge \quad y_1 = x_1 + 10 \quad \wedge \quad y_2 = y_1 + 10 \quad \wedge \quad y_2 \neq 30$$

$$\Rightarrow x_1 = 10$$

Predicate Refinement for Paths

Recall the decision problem we build for simulating paths:

$$x_1 = 10 \quad \wedge \quad y_1 = x_1 + 10 \quad \wedge \quad y_2 = y_1 + 10 \quad \wedge \quad y_2 \neq 30$$

$$\Rightarrow x_1 = 10 \qquad \Rightarrow y_1 = 20$$

Predicate Refinement for Paths

Recall the decision problem we build for simulating paths:

$$x_1 = 10 \quad \wedge \quad y_1 = x_1 + 10 \quad \wedge \quad y_2 = y_1 + 10 \quad \wedge \quad y_2 \neq 30$$

$$\Rightarrow x_1 = 10 \qquad \Rightarrow y_1 = 20 \qquad \Rightarrow y_2 = 30$$

Predicate Refinement for Paths

Recall the decision problem we build for simulating paths:

$$x_1 = 10 \quad \wedge \quad y_1 = x_1 + 10 \quad \wedge \quad y_2 = y_1 + 10 \quad \wedge \quad y_2 \neq 30$$

$$\Rightarrow x_1 = 10 \qquad \Rightarrow y_1 = 20 \qquad \Rightarrow y_2 = 30 \qquad \Rightarrow \text{false}$$

Predicate Refinement for Paths

Recall the decision problem we build for simulating paths:

$$\overbrace{x_1 = 10}^{A_1} \quad \wedge \quad \overbrace{y_1 = x_1 + 10}^{A_2} \quad \wedge \quad \overbrace{y_2 = y_1 + 10}^{A_3} \quad \wedge \quad \overbrace{y_2 \neq 30}^{A_4}$$

$$\Rightarrow x_1 = 10 \qquad \Rightarrow y_1 = 20 \qquad \Rightarrow y_2 = 30 \qquad \Rightarrow \text{false}$$

SATABS

Recall the decision problem we build for simulating paths:

$$\underbrace{x_1 = 10}_{\Rightarrow \underbrace{x_1 = 10}_{A'_1}}^{A_1} \quad \wedge \quad \overbrace{y_1 = x_1 + 10}^{A_2} \quad \wedge \quad \overbrace{y_2 = y_1 + 10}^{A_3} \quad \wedge \quad \overbrace{y_2 \neq 30}^{A_4}$$

$$\Rightarrow \underbrace{x_1 = 10}_{A'_1} \qquad \Rightarrow \underbrace{y_1 = 20}_{A'_2} \qquad \Rightarrow \underbrace{y_2 = 30}_{A'_3} \qquad \Rightarrow \underbrace{\text{false}}_{A'_4}$$

Predicate Refinement for Paths

SATABS

For a path with n steps:

$$\begin{array}{cc|cc|cc|cc|cc}
 & A_1 & & A_2 & & A_3 & & \cdots & & A_n & \\
\text{true} & & \Rightarrow A'_1 & & \Rightarrow A'_2 & & \Rightarrow A'_3 & & \Rightarrow A'_{n-1} & & \Rightarrow \text{false}
\end{array}$$

Predicate Refinement for Paths

SATABS

For a path with n steps:

$$
\begin{array}{cccccccccc}
 & A_1 & \vdots & A_2 & \vdots & A_3 & \vdots & \cdots & \vdots & A_n \\
\text{true} & & \Rightarrow A'_1 & & \Rightarrow A'_2 & & \Rightarrow A'_3 & & \Rightarrow A'_{n-1} & & \Rightarrow \text{false}
\end{array}
$$

- Given $A_1, \ldots, A_n$ with $\bigwedge_i A_i =$ false
- $A'_0 =$ true and $A'_n =$ false
- $(A'_{i-1} \wedge A_i) \Rightarrow A'_i$ for $i \in \{1, \ldots, n\}$

Predicate Refinement for Paths

For a path with n steps:

$$
\begin{array}{ccccccccccc}
 & A_1 & \vdots & A_2 & \vdots & A_3 & \vdots & \cdots & \vdots & A_n & \\
\text{true} & & \Rightarrow A'_1 & & \Rightarrow A'_2 & & \Rightarrow A'_3 & & \Rightarrow A'_{n-1} & & \Rightarrow \text{false}
\end{array}
$$

- Given $A_1, \ldots, A_n$ with $\bigwedge_i A_i =$ false
- $A'_0 =$ true and $A'_n =$ false
- $(A'_{i-1} \wedge A_i) \Rightarrow A'_i$ for $i \in \{1, \ldots, n\}$
- Finally, $Vars(A'_i) \subseteq (Vars(A_1 \ldots A_i) \cap Vars(A_{i+1} \ldots A_n))$

Predicate Refinement for Paths

Special case $n = 2$:

- $A \wedge B =$ false
- $A \Rightarrow A'$
- $A' \wedge B =$ false
- $Vars(A') \subseteq (Vars(A) \cap Vars(B))$

Predicate Refinement for Paths

SATABS

Special case $n = 2$:

- $A \wedge B =$ false
- $A \Rightarrow A'$
- $A' \wedge B =$ false
- $\mathit{Vars}(A') \subseteq (\mathit{Vars}(A) \cap \mathit{Vars}(B))$

W. Craig's Interpolation theorem (1957): such an A' exists for any first-order, inconsistent A and B.

Predicate Refinement with Craig Interpolants

SATABS

- ✔ For propositional logic, a propositional Craig Interpolant can be extracted from a resolution proof ($\rightarrow$ SAT!) in linear time
- ✔ Interpolating solvers available for linear arithmetic over the reals and integer difference logic with uninterpreted functions
- ✘ Not possible for every fragment of FOL:

$$x = 2y \quad \text{and} \quad x = 2z + 1 \qquad \text{with } x, y, z \in \mathbb{Z}$$

Predicate Refinement with Craig Interpolants

SATABS

- ✔ For propositional logic, a propositional Craig Interpolant can be extracted from a resolution proof ($\rightarrow$ SAT!) in linear time

- ✔ Interpolating solvers available for linear arithmetic over the reals and integer difference logic with uninterpreted functions

- ✘ Not possible for every fragment of FOL:

$$x = 2y \quad \text{and} \quad x = 2z + 1 \qquad \text{with } x, y, z \in \mathbb{Z}$$

The interpolant is "x is even"

Craig Interpolation for Linear Inequalities

$$\frac{0 \leq x \quad 0 \leq y}{0 \leq c_1 x + c_2 y} \quad \text{with } 0 \leq c_1, c_2$$

- “Cutting-planes”
- Naturally arise in Fourier-Motzkin or Simplex

Example

SATABS

$A = (0 \leq x - y) \wedge (0 \leq y - z - 1)$ $\quad$ $B = (0 \leq z - x)$

Example

$A = (0 \leq x - y) \wedge (0 \leq y - z - 1)$

$B = (0 \leq z - x)$

$0 \leq y - z - 1$

$0 \leq z - x$

Example

$A = (0 \leq x - y) \wedge (0 \leq y - z - 1)$ $\quad$ $B = (0 \leq z - x)$

$$\frac{0 \leq y - z - 1 \qquad 0 \leq z - x}{0 \leq y - x - 1}$$

Example

$$A = (0 \le x - y) \wedge (0 \le y - z - 1) \qquad B = (0 \le z - x)$$

$$\dfrac{\dfrac{0 \le y - z - 1 \qquad 0 \le z - x}{0 \le y - x - 1} \qquad 0 \le x - y}{0 \le -1}$$

Example

$$A = (0 \le x - y) \wedge (0 \le y - z - 1) \qquad B = (0 \le z - x)$$

$$\frac{\dfrac{0 \le y - z - 1 \qquad 0 \le z - x}{0 \le y - x - 1} \qquad 0 \le x - y}{0 \le -1}$$

$$\frac{\dfrac{0 \le y - z - 1 \qquad 0 \le \mathbf{0}}{0 \le y - z - 1} \qquad 0 \le x - y}{0 \le x - z - 1}$$

Example

$$A = (0 \leq x - y) \wedge (0 \leq y - z - 1) \qquad B = (0 \leq z - x)$$

$$\dfrac{\dfrac{0 \leq y - z - 1 \qquad 0 \leq z - x}{0 \leq y - x - 1} \qquad 0 \leq x - y}{0 \leq -1}$$

$$\dfrac{\dfrac{0 \leq y - z - 1 \qquad 0 \leq 0}{0 \leq y - z - 1} \qquad 0 \leq x - y}{0 \leq x - z - 1}$$

$$\iff \quad z - x \leq -1$$

Example

SATABS

$$A = (0 \le x - y) \wedge (0 \le y - z - 1) \qquad B = (0 \le z - x)$$

$$\dfrac{\dfrac{0 \le y - z - 1 \qquad 0 \le z - x}{0 \le y - x - 1} \qquad 0 \le x - y}{0 \le -1}$$

$$\dfrac{\dfrac{0 \le y - z - 1 \qquad 0 \le \mathbf{0}}{0 \le y - z - 1} \qquad 0 \le x - y}{0 \le x - z - 1}$$

$$\iff \quad z - x \le -1$$

Just sum the inequalities from A, and you get an interpolant!

Approximating Loop Invariants: SP

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The SP refinement results in

$$sp(\texttt{x=y=0}, \mathsf{true}) \quad = \quad x = 0 \wedge y = 0$$

Approximating Loop Invariants: SP

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The SP refinement results in

$$\begin{aligned} sp(\text{x=y=0}, \text{true}) &= x = 0 \wedge y = 0 \\ sp(\text{x++; y++}, \ldots) &= x = 1 \wedge y = 1 \end{aligned}$$

Approximating Loop Invariants: SP

SATABS

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The SP refinement results in

$$
\begin{aligned}
sp(\text{x=y=0}, \text{true}) &= x = 0 \land y = 0 \\
sp(\text{x++; y++}, \ldots) &= x = 1 \land y = 1 \\
sp(\text{x++; y++}, \ldots) &= x = 2 \land y = 2
\end{aligned}
$$

Approximating Loop Invariants: SP

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The SP refinement results in

$$
\begin{array}{lcl}
sp(\text{x=y=0}, \text{true}) & = & x = 0 \wedge y = 0 \\
sp(\text{x++; y++}, \ldots) & = & x = 1 \wedge y = 1 \\
sp(\text{x++; y++}, \ldots) & = & x = 2 \wedge y = 2 \\
sp(\text{x++; y++}, \ldots) & = & x = 3 \wedge y = 3 \\
\ldots & &
\end{array}
$$

✘ 10 iterations required to prove the property.

✘ It won't work if we replace 10 by n.

Approximating Loop Invariants: WP

SATABS

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The WP refinement results in

$$wp(\texttt{x==10}, y \neq 10) \quad = \quad y \neq 10 \wedge x = 10$$

Approximating Loop Invariants: WP

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The WP refinement results in

$$
\begin{aligned}
wp(\texttt{x==10}, y \neq 10) &= y \neq 10 \wedge x = 10 \\
wp(\texttt{x++; y++}, \ldots) &= y \neq 9 \wedge x = 9
\end{aligned}
$$

Approximating Loop Invariants: WP

SATABS

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The WP refinement results in

$$
\begin{aligned}
wp(\text{x==10}, y \neq 10) &= y \neq 10 \wedge x = 10 \\
wp(\text{x++; y++}, \ldots) &= y \neq 9 \wedge x = 9 \\
wp(\text{x++; y++}, \ldots) &= y \neq 8 \wedge x = 8
\end{aligned}
$$

Approximating Loop Invariants: WP

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The WP refinement results in

$$
\begin{aligned}
wp(\texttt{x==10}, y \neq 10) &= y \neq 10 \wedge x = 10 \\
wp(\texttt{x++; y++}, \ldots) &= y \neq 9 \wedge x = 9 \\
wp(\texttt{x++; y++}, \ldots) &= y \neq 8 \wedge x = 8 \\
wp(\texttt{x++; y++}, \ldots) &= y \neq 7 \wedge x = 7
\end{aligned}
$$

Approximating Loop Invariants: WP

SATABS

```
int x, y;

x=y=0;

while(x!=10) {
  x++;
  y++;
}

assert(y==10);
```

The WP refinement results in

$$
\begin{array}{lcl}
wp(\text{x==10}, y \neq 10) & = & y \neq 10 \wedge x = 10 \\
wp(\text{x++; y++}, \ldots) & = & y \neq 9 \wedge x = 9 \\
wp(\text{x++; y++}, \ldots) & = & y \neq 8 \wedge x = 8 \\
wp(\text{x++; y++}, \ldots) & = & y \neq 7 \wedge x = 7 \\
\ldots & &
\end{array}
$$

✘ Also requires 10 iterations.

✘ It won't work if we replace 10 by n.

What do we really need?

SATABS

Consider an SSA-unwinding with 3 loop iterations:

$$x_1 = 0$$
$$y_1 = 0$$

What do we really need?

Consider an SSA-unwinding with 3 loop iterations:

$$
\begin{array}{l|c}
 & \text{1st It.} \\
\begin{array}{l} x_1 = 0 \\ y_1 = 0 \end{array} &
\begin{array}{c} x_1 \neq 10 \\ x_2 = x_1 + 1 \\ y_2 = y_1 + 1 \end{array}
\end{array}
$$

What do we really need?

Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1 + 1$ $y_2 = y_1 + 1$	$x_2 \neq 10$ $x_3 = x_2 + 1$ $y_3 = y_2 + 1$

What do we really need?

Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.	3rd It.
	$x_1 \neq 10$	$x_2 \neq 10$	$x_3 \neq 10$
$x_1 = 0$	$x_2 = x_1 + 1$	$x_3 = x_2 + 1$	$x_4 = x_3 + 1$
$y_1 = 0$	$y_2 = y_1 + 1$	$y_3 = y_2 + 1$	$y_4 = y_3 + 1$

What do we really need?

SATABS

Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1+1$ $y_2 = y_1+1$	$x_2 \neq 10$ $x_3 = x_2+1$ $y_3 = y_2+1$	$x_3 \neq 10$ $x_4 = x_3+1$ $y_4 = y_3+1$	$x_4 = 10$ $y_4 \neq 10$

What do we really need?

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Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1 + 1$ $y_2 = y_1 + 1$	$x_2 \neq 10$ $x_3 = x_2 + 1$ $y_3 = y_2 + 1$	$x_3 \neq 10$ $x_4 = x_3 + 1$ $y_4 = y_3 + 1$	$x_4 = 10$ $y_4 \neq 10$

$$x_1 = 0$$
$$y_1 = 0$$

What do we really need?

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Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1 + 1$ $y_2 = y_1 + 1$	$x_2 \neq 10$ $x_3 = x_2 + 1$ $y_3 = y_2 + 1$	$x_3 \neq 10$ $x_4 = x_3 + 1$ $y_4 = y_3 + 1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$ $\quad$ $x_2 = 1$

$y_1 = 0$ $\quad$ $y_2 = 1$

What do we really need?

SATABS

Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1 + 1$ $y_2 = y_1 + 1$	$x_2 \neq 10$ $x_3 = x_2 + 1$ $y_3 = y_2 + 1$	$x_3 \neq 10$ $x_4 = x_3 + 1$ $y_4 = y_3 + 1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$, $y_1 = 0$ $\qquad$ $x_2 = 1$, $y_2 = 1$ $\qquad$ $x_3 = 2$, $y_3 = 2$

What do we really need?

SATABS

Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1+1$ $y_2 = y_1+1$	$x_2 \neq 10$ $x_3 = x_2+1$ $y_3 = y_2+1$	$x_3 \neq 10$ $x_4 = x_3+1$ $y_4 = y_3+1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$, $y_1 = 0$ | $x_2 = 1$, $y_2 = 1$ | $x_3 = 2$, $y_3 = 2$ | $x_4 = 3$, $y_4 = 3$

What do we really need?

Consider an SSA-unwinding with 3 loop iterations:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1{+}1$ $y_2 = y_1{+}1$	$x_2 \neq 10$ $x_3 = x_2{+}1$ $y_3 = y_2{+}1$	$x_3 \neq 10$ $x_4 = x_3{+}1$ $y_4 = y_3{+}1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$, $y_1 = 0$ $\quad$ $x_2 = 1$, $y_2 = 1$ $\quad$ $x_3 = 2$, $y_3 = 2$ $\quad$ $x_4 = 3$, $y_4 = 3$

✘ *This proof will produce the same predicates as SP.*

Split Provers

Idea:

$$\mathcal{P}_1 \rightleftarrows \mathcal{P}_2 \rightleftarrows \mathcal{P}_3 \rightleftarrows \cdots \rightleftarrows \mathcal{P}_n$$

- Each prover $\mathcal{P}_i$ only knows A_i, but they exchange facts
- We require that each prover only exchanges facts with common symbols
- Plus, we restrict the facts exchanged to some language $\mathcal{L}$

Back to the Example

SATABS

Restriction to language $\mathcal{L} =$ “no new constants”:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1 + 1$ $y_2 = y_1 + 1$	$x_2 \neq 10$ $x_3 = x_2 + 1$ $y_3 = y_2 + 1$	$x_3 \neq 10$ $x_4 = x_3 + 1$ $y_4 = y_3 + 1$	$x_4 = 10$ $y_4 \neq 10$

Back to the Example

Restriction to language $\mathcal{L}$ = "no new constants":

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1 + 1$ $y_2 = y_1 + 1$	$x_2 \neq 10$ $x_3 = x_2 + 1$ $y_3 = y_2 + 1$	$x_3 \neq 10$ $x_4 = x_3 + 1$ $y_4 = y_3 + 1$	$x_4 = 10$ $y_4 \neq 10$

$$x_1 = 0$$
$$y_1 = 0$$

Back to the Example

SATABS

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$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1 + 1$ $y_2 = y_1 + 1$	$x_2 \neq 10$ $x_3 = x_2 + 1$ $y_3 = y_2 + 1$	$x_3 \neq 10$ $x_4 = x_3 + 1$ $y_4 = y_3 + 1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$ $\quad$ $x_2 = 1$

$y_1 = 0$ $\quad$ $y_2 = 1$

Back to the Example

Restriction to language $\mathcal{L}$ = “no new constants”:

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1+1$ $y_2 = y_1+1$	$x_2 \neq 10$ $x_3 = x_2+1$ $y_3 = y_2+1$	$x_3 \neq 10$ $x_4 = x_3+1$ $y_4 = y_3+1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$, $y_1 = 0$ $\quad$ $x_2 = 1$, $y_2 = 1$ $\quad$ $x_3 = 2$, $y_3 = 2$

Back to the Example

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Restriction to language $\mathcal{L}$ = "no new constants":

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$x_1 = 0$ $\quad$ $y_1 = 0$

$x_2 = 1$ $\quad$ $y_2 = 1$

$x_3 = 2$ $\quad$ $y_3 = 2$

Back to the Example

SATABS

Restriction to language $\mathcal{L}$ = "no new constants":

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1+1$ $y_2 = y_1+1$	$x_2 \neq 10$ $x_3 = x_2+1$ $y_3 = y_2+1$	$x_3 \neq 10$ $x_4 = x_3+1$ $y_4 = y_3+1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$, $y_1 = 0$ $\qquad$ $x_2 = 1$, $y_2 = 1$ $\qquad$ $x_3 = y_3$

Back to the Example

SATABS

Restriction to language $\mathcal{L}$ = "no new constants":

	1st It.	2nd It.	3rd It.	Assertion
$x_1 = 0$ $y_1 = 0$	$x_1 \neq 10$ $x_2 = x_1{+}1$ $y_2 = y_1{+}1$	$x_2 \neq 10$ $x_3 = x_2{+}1$ $y_3 = y_2{+}1$	$x_3 \neq 10$ $x_4 = x_3{+}1$ $y_4 = y_3{+}1$	$x_4 = 10$ $y_4 \neq 10$

$x_1 = 0$, $y_1 = 0$ $\qquad$ $x_2 = 1$, $y_2 = 1$ $\qquad$ $x_3 = y_3$ $\qquad$ $x_4 = y_4$

Invariants from Restricted Proofs

SATABS

✔ The language restriction forces the solver to generalize!

- Algorithm:
  - If the proof fails, increase $\mathcal{L}$!
  - If we fail to get a sufficiently strong invariant, increase n.

✔ This does work if we replace 10 by n!

Invariants from Restricted Proofs

SATABS

✔ The language restriction forces the solver to generalize!

- Algorithm:

  - If the proof fails, increase $\mathcal{L}$!
  - If we fail to get a sufficiently strong invariant, increase n.

✔ This does work if we replace 10 by n!

? Which $\mathcal{L}_1, \mathcal{L}_2, \ldots$ is complete for which programs?